CONCRETE-REPRESENTATIONAL-ABSTRACT SEQUENCE OF INSTRUCTION IN GEOMETRY: AN INTERVENTION FOR LOW PERFORMING EIGHT GRADERS

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Abstract

This study employed a quasi-experimental design that aimed to determine the effectiveness of Concrete-Representational-Abstract (CRA) sequence of instruction in Geometry as an intervention for low performing eighth graders.

Two equivalent groups with 34 Grade 8 students in each were purposively organized to serve as the control and experimental groups. Standardized pre and post tests were the primary instruments used in evaluating the students' performances. The experimental group received instruction on Triangle Congruence with the use of CRA while the control group was taught the same subject matter in the conventional way.

Pretest results showed that the control and experimental groups have almost the same level of initial knowledge of the lesson while the posttest scores' comparison between the two groups showed a significant difference which may be attributed to the use of CRA as an intervention. Thus, the study recommends the application of CRA in teaching geometry and other Mathematics subjects in order to further establish the usefulness of the intervention in helping struggling students in Geometry.

Keywords: Concrete-Representational-Abstract (CRA), geometry, low performers in mathematics, grade 8 mathematics, teaching mathematics

INTRODUCTION

Mathematics is one of the core subjects in the Philippine education in which students find difficulty (Chua, 2006; Pangan, 2010). Studies have shown that there are many factors affecting students' achievement in Mathematics (Mahanta, 2012).

Mathematics is indispensible as individuals perform tasks that involve it; hence, priority may need to be given to this subject especially in the secondary level where basic concepts of Geometry and Algebra are taught to high school students, and these branches of Mathematics later become the foundation for higher Mathematics in tertiary education. Likewise, there is a need for teachers to upgrade and improve their teaching strategies to become parallel with the learning styles of the students along with their experiences, reflections, conceptualizations, and experimentations. High school teachers may employ different techniques in experiential learning in the classroom teaching-learning process (Albay, 2009).

As stated in Batanero and Diaz (2011), the Philippines falls behind other Asian countries when it comes to quality education. One of the reasons for the poor performance is the low quality of basic education in the country. In May 2010, a test that was administered to all grade six graduates in public elementary schools reflected very low scores in Science and Mathematics tests.

According to Canonizado (2009), students learn Mathematics through the experiences that teachers provide. Giving daily practice to pupils in the four basic operations, having a dialogue with parents every month about their children's progress, fund raising for the giving of incentives to children, giving daily formative tests and assignments, monitoring strictly the pupil's attendance and requiring children to read and study their lessons during their free time would likely help them excel in Mathematics subjects (Jubelag, 2009).

Many people consider Mathematics as a key in opening future career options. Mathematics is in the middle of today's reform efforts that intend to establish a system for guiding students in their learning and understanding of mathematics (Arslan, 2012).

The National Curriculum Framework (NCF 2005) also categorically stressed the importance of mathematics and opined that "a high quality mathematics program is essential for all students and it provides every student with the opportunity to choose among the full range of future career path" (P.96).

Furthermore, the teaching of Geometry in the Philippines is part of the Mathematics curricula for elementary and secondary schools and thus, handled by mathematics teachers. The word 'geometry' comes from two ancient Greek words: *geo* means earth and *metron* meaning to measure. As Jones (2002) pointed out, one of the wonderful branches of mathematics to teach is Geometry. It is full of interesting problems and astonishing theorem that appeals to the visual, aesthetic and intuitive senses.

Studying Geometry is one of the major vehicles to develop the skills of the learners in visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proving stances. Moreover, for one to help students make sense of other areas of mathematics such as in dealing with fractions and multiplication in arithmetic, in understanding the relationships between the graphs of functions (of both two and three variables), and graphical representations of data in statistics, geometry may be used. Geometric representation may also be applied in managing all the aspects of daily lives. Fine motor skills can be developed by using spatial reasoning in other field of specialization such as mathematics, geography, art design and technology (Jones, 2002).

Most of the learners today might have difficulties in understanding the lesson in Mathematics especially in Geometry because of many reasons. These reasons might include the ways on how the teachers teach the lessons in class. In Sousa (2008) the Concrete-Representational-Abstract (CRA) was used in order to help students who have difficulties in understanding the lesson in Mathematics. It is a manipulative technique. This instructional approach helps students create meaningful connections among concrete, representational, and abstract levels of thinking and understanding which stems from its graduated, conceptually supported framework. It has been shown that the CRA sequence was mostly effective with learners who have difficulties in Mathematics.

In a recent study on using the CRA instruction sequence in teaching subtraction with regrouping to some low - achieving Grade 3 mathematics students, Flores (2010) reported that students show improvement in fluency and confidence in doing arithmetic computations involving subtractions. In addition, a number of other studies have provided evidence of positive effects of using CRA on low achievers in the area of fractions (Butler, Miller, Krehan, Babbitt & Pierce, 2003), word problems (Maccini, Mulcahy & Wilson, 2007), simple linear functions (Witzel, 2005), and advanced linear functions (Witzel, Mercer & Miller, 2003). Indeed, the use of CRA approach to teaching mathematics concepts, especially at the elementary level has been proven to be effective.

Acquiring a strong foundation on academic math skills like addition and multiplication at an early age contributes significantly to the success in later math-related courses, especially in tertiary education (Fletcher, Boon & Cihak, 2010). However, despite this positive turnout, an issue arise on children leaving school having underachieved in mathematics leading to their becoming innumerate individuals (Rashid & Brooks, 2010).

The Concrete-Representational-Abstract sequence was also used to enhance the mathematics performance of the students. It help students involves the use of three distinct teaching phases with students showing mastery at each phase prior to moving to the next (Miller & Hudson, 2007).

Scheuermann, Deshler and Schumaker (2009) conducted a study designed to explore the CRA instructional sequence through explicit instruction while solving word problems. The purpose of the study was to investigate the effectiveness of this approach in both general education and special education settings. The researchers concluded that students with mathematics learning disabilities can increase their knowledge of mathematical concepts using direct instruction and the CRA instructional sequence.

The concrete-representational-abstract (CRA) sequence of instruction is the most common example of mathematics instruction incorporating visual representations. The CRA technique actually refers to a simple concept that has proven to be a highly effective method of teaching mathematics to students with disabilities (Steedly, 2008).

As Steedly (2008) pointed out, CRA works well with individual students, in small groups, and with an entire class. It is also appropriate at both the elementary and secondary levels. When using CRA, teachers must make sure that students understand what has been taught at each step before moving instruction to the next stage.

In CRA, the learning of the students starts with visual, concrete, and kinesthetic experiences in order to establish basic understanding. Once accomplished, students will then be able to extend their knowledge by means of representations of pictures. Finally, they will become independent from the tangible objects and will be able to comprehend the abstract mathematical concepts (Hudson, Miller, & Butler, 2006).

Figure 1 shows the paradigm of the study. The conceptual framework of this study was patterned after a descriptive map which was also called thematic map (Witzel, Mercer, & Miller, 2003). The paradigm theorizes that the Concrete-Representational-Abstract (CRA) is potent in creating an impact to improve students' performance in Geometry.

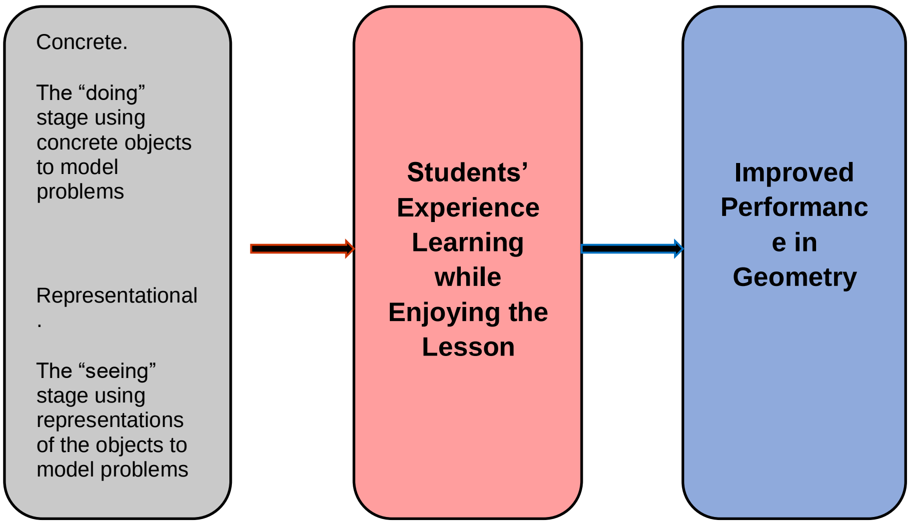

Figure 1: Paradigm of the Study

The main goal of the present study was to search for relevant teaching strategies to improve the performance of low performing eighth graders in Geometry and subsequently strengthen their analytical and reasoning skills. Among the creative means of teaching geometrical concepts and skills that mathematics experts recommend is the Concrete-Representational-Abstract (CRA) sequence of instruction which this study investigated on through experimentation. Specifically, it sought to determine (1) the mathematical performance of the respondents before and after the use of conventional learning for the control group and concrete-representational-abstract sequence of instruction for the experimental group, and (2) if there is significant difference in the mathematical performance of the respondents between the control and experimental groups before and after the instruction.

It is hoped that the results of this investigation would encourage teachers to explore the use of the strategy in their respective classes in order to boost students' performance in Geometry and other relevant subject areas. It is also desired that researchers and curriculum developers derive inputs from this study that may assist them in improving instructional materials and devising new strategies for Geometry teachers.

METHOD

The quasi-experimental method of research was used in the study in order to gather evidence of the effectiveness of Concrete-Representational-Abstract (CRA) technique. Quasi-experimental design involves selecting groups, upon which a variable is tested, without a random pre-selection purposes.

Quasi-experiments have been used as far back as the 18th century and continued to be frequently utilized by researchers today for three primary reasons: (1) to meet the practical requirements of funding, school administrators, and ethics, (2) to evaluate the effectiveness of an intervention when the intervention has been implemented by educators prior to the evaluation procedure having been considered and (3) when researchers want to dedicate greater resources to issues of external and construct validity (Shadish, Cook & Campbell, 2002).

This research design includes the use of pretest and posttest for both the control and experimental groups. The quasi-experimental design was considered as the most relevant and applicable method to be used since the respondents were pre-qualified.

The respondents were the heterogeneously sectioned grade eight students of a public high school, who were taking Geometry lessons for the third grading period. They were the two sections in grade eight. Lottery sampling was used in choosing the group which will be assigned as control and experimental. Afterwards, purposive sampling was used in choosing the respondents in the experimental and control groups: those who qualified as lower performers were those who obtained a final grade below 80.

Since the number of the students who obtained a final grade below 80 in the control group was 34 and the number of students who obtained the same final grade for the experimental group was 37, there was a need to remove three (3) students. This was carried out using the lottery sampling in order to avoid bias in excluding the excess number of respondents.

To conceal the identity of the respondents, they were assigned consecutive codes (example: Student C1 for the first student in the control group; Student E1 for the student in the experimental group). The list of the respondents' names was requested from the advisers of each section.

The primary instrument that was used for the gathering of data was adopted from the standardized twenty-item test of the Department of Education Grade Eight Learners' Module. The test paper was a multiple-choice type of test. The content of the test was about the triangle congruence postulates, combinations of congruent sides and angles that are used to determine if the two triangles are congruent.

The lesson plan was prepared using the module provided by the Department of Education twisted with the CRA Approach as a guide. The lesson plan describes combinations of discovering congruent triangles and triangle congruence postulates.

Prior to the conduct of the study, permission was formally sought from the school principal. Grouping took place upon the approval of the school authority and of the concerned teachers (class advisers). Then the actual experimentation commenced when grouping was settled. Step one consisted of the administration of pre-test to both groups and recording of results. This was followed by the teaching of the two groups.

The concrete-representational-abstract approach in teaching triangle congruence was used to the experimental group using the lesson plan that was developed. Concrete components include manipulatives, measuring tools, or other objects the students can handle during the lesson, representational components includes tables, charts, diagrams that represent the data, and abstract components include symbols and numbers (Witzel, Mercer & Miller, 2003).

Meanwhile, the conventional method of teaching Mathematics was used for the control group. The lesson was carried out for seventeen (17) school days, the same number of days as in the experimental group. This is to ensure that parallel conditions are provided to both groups except for the intervention under study. The administration of the post-test for both the experimental and the control groups was made after the instruction. All the data were analyzed and interpreted based on the objectives of the study.

The following statistical tools were used to analyze and interpret the quantitative data: the Mean formula was used to determine the arithmetic average among the pre and post-test results of the respondents; T-test for Independent Sample Mean was used to determine if there was significant difference between the experimental and control groups' pre and posttest scores.

The results of the Pre-and Post-Assessment were organized, analyzed, and interpreted using the following arbitrary scale:

Raw Scores	**Descriptive Rating**
15.50 – 20.00	Excellent (E)
10.50 – 15.49	Satisfactory (S)
5.50 – 10.49	Fair (Fa)
0 – 5.49	Failed (F)

RESULTS

Respondents' Mathematical Performance Before and After the Instruction

Table 1 shows the mathematical performance of both the control and experimental groups before and after the use of conventional learning and concrete-representational-abstract sequence of instruction in teaching geometry.

Pre-test results reflect that, the grand mean of the experimental group was 5.53 which was interpreted as Fair (Fa) with standard deviation of 1.91 while the grand mean of the control group was 5.06 which was interpreted as Failed (F) with standard deviation of 1.37.

After the instruction, the experimental group got a grand mean score of 16.44 which was interpreted as Excellent (E) with standard deviation of 2.16. On the other hand, the control group got a grand mean score of 13.79 which was interpreted as Satisfactory (S) with standard deviation of 1.63.

The concept of standard deviation is especially valuable because it compares the data from different sets of data. When two groups are compared, the group having a smaller standard deviation is less varied. The results therefore show that the scores of the experimental group varies more than their counterparts in the control group.

Table 1
Respondents' mathematical performance before and after the instruction

(Before the Instruction)

Group	Score	No. of Student	Mean Score	Descriptive Rating	Standard Deviation
Experimental Group	15.50-20.00 (E) 10.50 – 15.49 (S) 5.50 – 10.49 (Fa) 0 - 5.49 (F)	0 0 16 18	5.53	Fair	1.91
Control Group	15.50-20.00 (E) 10.50 – 15.49 (S) 5.50 – 10.49 (Fa) 0 - 5.49 (F)	0 0 12 22	5.06	Failed	1.37

(After the Instruction)

Group	Score	No. of Student	Mean Score	Descriptive Rating	Standard Deviation
Experimental Group	15.50-20.00 (E) 10.50 – 15.49 (S) 5.50 – 10.49 (Fa) 0 - 5.49 (F)	21 13 0 0	16.44	Excellent	2.16
Control Group	15.50-20.00 (E) 10.50 – 15.49 (S) 5.50 – 10.49 (Fa) 0 - 5.49 (F)	4 28 2 0	13.79	Satisfactory	1.63

Difference between the Two Groups' Pretest Results

The pre-test results of the experimental and control groups are presented in Table 2. Using t-test of independent samples at 0.05 level of significance, the t-value obtained was 1.08 with a p-value of 0.287, respectively, showing no significant difference. This seems to show that there is no significant difference between the pre-test result of the experimental and control groups showing that students in the two groups have the same level of knowledge in geometry prior to the instruction using CRA sequence of instruction.

Table 2
Difference between the two groups' Pretest Results

Groups	Means (PRE-TEST)	Computed t-value	Critical Value	p-value
Experimental Group	5.53	1.08	1.99	0.287 (not significant at 0.05 level)
Control Group	5.06			

Difference between the Two Groups' Posttest Results

Seen in Table 3 is the difference between the post-test results of the experimental and control groups. Using t-test of independent samples at 0.05 level of significance, the value obtained was 5.23 with the p-value of 0.00, respectively, showing significant difference.

Table 3
Difference between the Two Groups' Posttest Results

Groups	Means (POST-TEST)	Computed t-value	Critical Value	p-value
Experimental Group	16.44	5.23	1.99	0.00 (Significant at 0.05 level)
Control Group	13.79			

Difference in the Incremental Scores of the Experimental Group and Control Group

The results of the two groups' incremental scores are presented in Table 2. Using t-test of independent samples at 0.05 level of significance, the p-value of 0.00 and mean incremental scores are 10.91 for the experimental group and 8.73 for the control group, respectively, showing significant difference.

Table 4
Results of the two groups' incremental scores

Groups	No. of Respondents	Mean Incremental Scores	Standard Deviation	t-test p-value
Experimental Group	34	10.91	2.04	0.00 (Significant at 0.05 level)
Control Group	34	8.73	1.50	

DISCUSSION

Chua, as cited in Pangan (2010) found that the National Achievement Test (NAT) results for secondary level is as low as 46.38% while the Trends in International Mathematics and Science Study (TIMSS) results have shown that the country ranked 39th in Mathematics out of 42 participating countries in 1995, ranked third to the last in the 1999 and placed 41st in among 42 participant countries in 2003. These results have drawn studies to look closely into the performance of students specifically in the two subject areas: science and mathematics. It is important to study the performance of the students in secondary education, particularly in mathematics which is considered to be a key factor for the development of science and technology as well as for the growth of the nation. It is also necessary to identify doable means for teachers to facilitate science and mathematics learning of students.

This paper aimed to contribute in searching for relevant and doable ways to faciliate the teaching of critical concepts in geometry, particularly in looking for teaching strategies that may improve the performance of low performing eighth graders in Geometry and subsequently strengthen their analytical and reasoning skills. Students' achievement is one of the main factors in determining the quality of education in the country. In identifying the factors that affect students' learning, achievement continues to be an important object of study of educators of different countries. Studies have shown that there are many factors affecting students' achievement in mathematics, including those that relate to attitude toward mathematics and opportunity to learn (Yaratan & Kasapo,2012), which are closely connected to classroom teaching and learning processes. Also, they are concerned with the tools, methods and approaches that facilitate practice or the study of practice toward enhancing the quality and performance of the students in Mathematics (Mahanta, 2012).

As drawn from the result of the study, it was revealed that there is a significant difference in the pretest results of the respondents which implies that both experimental and control groups have the same level of knowledge in the lesson. Based on the results of the posttest, it can be seen that there is a significant difference between the mean scores of the two groups. This implies the effectiveness of the concrete-representational-abstract sequence of instruction as a strategy to improve students' performance. Refering to previous similar studies, Flores (2010) also finds that the use of the CRA instructional sequence yielded positive results in improving students' performance, thus concludes on its effectiveness for teaching basic mathematics; it is effective for dealing with complete mathematical processes, which provides students greater access to the general education curriculum. Furthermore, students who used the CRA sequence of instruction performed fewer procedural errors in the lesson (Witzel, Mercer, & Miller, 2003).

Also, the mean incremental scores of the respondents show that there is a significant difference between the experimental and control groups. Outcome of the study confirms the concept of Witzel, Mercer, and Miller (2003) that Concrete-Representational-Abstract sequence of instruction can help motivate students and think actively in the teaching and learning process.

One of the wonderful branches of mathematics to teach is Geometry. It is full of interesting problems and astonishing theorems that appeal to the visual, aesthetic and intuitive senses. Geometry is part of most secondary school mathematics curricula under the K to 12 Program. The language of Geometry is seen to be a part of everyday discourse as it is used to create eye-catching headlines and sound-bites (Jones, 2002). This supports the reason why students need to engage with geometry in school.

Most of the learners today might have difficulties in understanding the lesson in Mathematics especially in Geometry because of many reasons. These reasons might include the ways on how the teachers teach the lessons in class. To address the difficulties and challenges for mathematics teachers in teaching Geometry, there is a need to examine teachers' own conceptions of Geometry and to consider discovering more effective approaches than the typical deductive methods of Mathematics instruction. One effective intervention for mathematics instruction that can enhance the mathematics performance of the students is the concrete-representational-abstract (CRA) sequence of instruction.

If Concrete Representational Abstract (CRA) sequence of instruction is used in teaching Geometry, it could bring about positive responses from the students – enjoyment, attention and engagement in class discussions and activities. Concrete-representational-abstract teaching sequence is a popular approach. This sequence involves the use of manipulatives, then gradually removes these aids until students become able to solve mathematical problems through the use of numbers or symbols only (Flores, Hinton, & Schwerk, 2014). On the other hand, Luke (2012) mentioned that problem solving can promote students' conceptual understanding and foster their ability to reason and communicate mathematically.

As a mathematics teacher, CRA approach was used to identify its impact to the students in Geometry. It is also a teaching strategy that is advocated by the Ministry of Education (Ministry of Education, 2012), embedded in textbooks used by schools (Fan, 2012), and taught in pre-service courses of mathematics teachers (e.g., Chua, 2010; Edge, 2006).

The instructional strategy recommended by Ketterlin-Geller, Chard and Fien (2008) is a gradated instructional sequence that proceeds from concrete to representational to abstract (CRA) benefits struggling students (p. 35).

The purpose of the concrete-representational-abstract approach is to make certain that learners develop a tangible understanding of the Mathematics skills or concepts that they learn (Special Connections, 2005). With the foundation that students acquire through CRA, they can later relate their conceptual understanding to abstract problems and learning

In the present study, the control group was taught Geometry applying the conventional way of teaching where the teacher discussed in front of the class with a chalkboard or an overhead projector. The students were sitting in straight rows looking at and attentively listening to the teacher, with a paper and pen ready to take notes and responding to the questions of the teacher during discussion.

Meanwhile, the Concrete Representational Abstract (CRA) sequence of instruction was used in the experimental group where students were made to accomplish task in small groups with supervision from the teacher. As Steedly (2008) pointed out, CRA works well with individual students, in small groups, and with an entire class. It is also appropriate at both the elementary and secondary levels. Studies have also shown that in lessons where manipulatives were used, students appeared to be interested, active, and involved in their learning, seeing math as a fun activity (Moyer, 2002). Good teaching involves communication and building relationships with students (Oppendekker & Van Damme, 2006).

Students used manipulative for the concrete stage in CRA to discuss among themselves and develop content. This is known as the “doing” stage that involves physically manipulated objects. The hands-on experience of manipulating objects is simple but this has been applied in a wide array of mathematical concepts and has brought forth promising/favorable results (Bouck et al., 2014).

Each math concept/skill was first modeled with concrete materials (e.g. straws, ruler, scissors, protractors, popsicle sticks). The teacher used materials which can easily be found or accessed. This was also for the sake of other teachers who may decide to try the same technique. In this manner, they will no longer have a hard time administering the activity. Students were given sufficient opportunities to practice and demonstrate mastery using concrete materials. Students were also allowed to construct understanding of mathematical concepts in social groups and through interaction with the teacher as a facilitator.

Manipulatives provide an opportunity of bridging the gap between the concrete and the abstract (Domino, 2010). On the other hand, additional factors must also be taken into consideration since students can also improve their skills in mathematics through manipulatives. Using manipulatives enables students to utilize their real-world practical knowledge (Rittle-Johnson & Koedinger, 2005) and prompt physical action, having been shown to enhance memory and comprehension (Martin & Schwartz, 2005). In addition, the mere incorporation of manipulatives into the teaching of mathematics may not suffice to increase achievement/performance (Carbonneau et al., 2013). As such, one must not assume that student will immediately understand the mathematical concepts or relationships by only interacting with objects. Hence, manipulatives should not be used as an ‘add on’, rather, their use must be explicitly explained and properly modeled to ensure understanding and comprehension.

Yagci (2010) probed the effect of mathematics instruction using concrete models on the achievement and attitudes of grade eight students. He likewise examined the perception of the students regarding instruction with concrete models. Students can solve concrete problems even if they are complicated. However, they find difficulty in abstract problems. Students' cognitive development improves at this stage. They are in need of activities including concrete models to acquire cognitive development.

The statistical results of Yagci showed that there was a statistically significant change in the achievement of the eighth grade students who were exposed to the instruction using concrete models or manipulatives. Additionally, it was identified that most of the students viewed the instruction with concrete models positively, thus bringing forth advantageous effects on their cognitive processes and on their attitudes toward concrete models.

Then the next step in CRA is the representational stage. It is known as the "seeing" stage where students exhibit mastery of skill by means of using concrete objects, by describing and modeling the skill by picture representation of manipulatives. The students are also provided with many opportunities to practice and demonstrate the mastery of skill.

The final phase in CRA is the abstract stage. It is known as the 'symbolic' stage, since it uses mathematical symbols in solving the problem. After the students successfully surpassed the representational level, the mathematics instructor guides the students in moving to the abstract level of understanding for a particular math or skill.

Thigpen (2012) discusses symbolic representation and its importance in students' ability to communicate their mathematical ideas to others. The symbols being used to communicate the mathematical idea of integer addition are numbers and mathematical signs, specifically positive and negative signs. The numbers and signs are the symbols of the mathematical concept, and students must obtain the knowledge on how to use these symbolic representations in order to be efficient in their mathematical endeavors.

Abstract concepts can be represented by manipulatives. As such, these abstract concepts represented visually foster understanding and provide a strategy that works for children who are struggling with mathematics (Carbonneau, Marley & Selig, 2013). Manipulatives and visual aids helped students to retain ability in understanding of the math concepts (Boggan, Harper & Whitmire, 2009).

The learning is active using CRA approach. Students are encouraged to stay focused and engaged throughout the learning process and retain the information and skills longer. Students were seen to be enjoying the activity. Students appeared to have built a better connection when moving through the levels of understanding from concrete to abstract. It also helped them learn concept before learning rules. The CRA is the most common example of mathematics instruction incorporating visual representations. It actually refers to a simple concept that has proven to be an effective method of teaching mathematics to students with disabilities (Steedly 2008). Students were observed to be more interactive, cooperative, engaged and collaborative.

Through this study/research, teachers in the area of mathematics may benefit a great deal from lessons having different models that approach/tackle a concept at different cognitive levels. The field of mathematics in education has recognized a significant number of studies showing that the ideal presentation sequence for new mathematical content is concrete-representational-abstract approach (CRA).

As stated in Sousa (2008), the CRA is beneficial to students who have difficulties in learning mathematical concepts. Various research studies have been conducted to support the effectiveness of this approach even in other areas of Mathematics. One of these is a study conducted by Witzel and his colleagues. They employed the CRA approach to sixth- and seventh-grade students who are mathematically-challenged in learning algebra. The students who learned how to solve algebraic equations through CRA approach achieved higher scores during the post-test compared to the control peers who received the traditional approach in teaching algebra. Consequently, students who used the CRA approach performed fewer errors in the process of solving algebraic expressions (Witzel, Mercer, & Miller, 2003).

With this, the CRA approach appears as one of the effective approaches in Mathematics that provide a clear picture of how students understand and learn mathematics. It also combines effective components of both behaviorist and constructivist practices. The direct instruction for the behaviorist and the discovery-learning for constructivist (Sealander et al., 2012).

Approaches such as CRA that contains concrete and pictorial representations of abstract concepts promote understanding and facilitate student success. Although manipulatives have been predominantly used in the elementary setting, this study finds that they are a valuable tool for instruction in the secondary setting as well. The CRA algebra model demonstrated generalized abstract concepts in a manner that facilitates students' active engagement as it adapts to each student's learning style.

For the mathematics students in the first and third graders, Fuchs and Hollenback (2007) also advocate the use of the CRA sequence to teach place value, Geometry, and fractions. Steedly and colleagues (2007) and Stein and colleagues (2006) concluded that effective teaching is not haphazard, but instead requires purposeful and systematic sequence of instruction. They agree that students should be taught new skills after mastering prerequisite skills. Additionally, effective instruction includes explicit instruction of skills that scaffold student learning. Student learning is enhanced through purposeful modelling, guided practice, and finally independent practice (Steedly et al., 2007; Stein et al., 2006).

Choudhury (2012) concluded that the student's attitude toward Mathematics affect their achievement. Moreover, the achievement in the subject Mathematics mostly depends on concept and practice. Parents and teachers agree that study habit influence pupil's achievement in Mathematics. Attitude towards Mathematics may depend mainly on the home environment and the parent's attitude toward Mathematics.

In reference to the foregoing conclusion, the following recommendations are hereby provided: Mathematics teachers should continuously propose instructional strategies and techniques that will be effective in helping more students learn and develop their skills and increase achievement on mathematical analysis. Writers of textbooks may consider the results of the study to modify instruction of particular topics using the characteristics of CRA approach. For future researchers, the same strategy may be tested involving an item analysis in the pre and posttest results to design more valid measures of determining the usefulness of CRA and eventually device CRA approaches that would further broaden the mastered skills with CRA. It is recommended also to have extensive research on the effectiveness of CRA in learning Mathematics concepts such as shifting the experiment into a series of tasks and comparing the progress of the students. In order to avoid bias, future researchers may involve randomly selected teachers who will be carrying out the lessons.

On the contrary, taking into account the difficulties students with learning disabilities' experience, it is imperative to integrate teaching techniques and approaches that are both effective and efficient to cater and be of help to high school students with learning disabilities in accessing the general education math curriculum in a meaningful manner (Maccini, Mulcacahy, & Wilson, 2007).

Thus, intensive research in mathematics teacher education is needed. There is increasing literature about relevant results, however, large-scale findings about the conditions, processes and effects of mathematics teacher education are rare (Adler et al. 2005). Since Mathematical Content Knowledge (MCK) and Mathematical Pedagogical Content Knowledge (MPCK) play a fundamental role for teachers' effectiveness (Baumert et al. 2010), the education of future teachers is a crucial phase in teachers' professional development and a key time for communicating pedagogical innovations, especially because many teachers tend to teach as they have been taught.

In this light, through the concrete-representational-abstract sequence of instruction, teaching Geometry is made easier in the teaching and learning process. Moreover, a high performance was observed from the students through mathematical analysis.

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